June 2012
6669 Further Pure Maths FP3
Mark Scheme

Question Number	Scheme	Marks
1. (a)	Uses formula to obtain $e=\frac{5}{4}$	M1A1
	Uses ae formula	M1 (3)
(b)	Uses other formula $\frac{a}{e}$	M1
	Obtains both Foci are $(\pm 5, 0)$ and Directrices are $x = \pm\frac{16}{5}$ (needs both method marks)	A1 cso (2) (5 marks)

Notes

a1M1: Uses $b^2 = a^2(e^2 - 1)$ to get $e > 1$

a1A1: cao

a2M1: Uses ae

b1M1: Uses $\frac{a}{e}$

b1A1: cso for both foci and both directrices. Must have both of the 2 previous M marks may be implicit.

Question Number	Scheme	Marks
2.	$\dfrac{dy}{dx} = \sinh 3x$	B1
	so $s = \int \sqrt{1+\sinh^2 3x}\,dx$	M1
	$\therefore s = \int \cosh 3x\,dx$	A1
	$= \left[\frac{1}{3}\sinh 3x\right]_0^{\ln a}$	M1
	$= \frac{1}{3}\sinh 3\ln a = \frac{1}{6}[e^{3\ln a} - e^{-3\ln a}]$	DM1
	$= \frac{1}{6}(a^3 - \dfrac{1}{a^3})$ (so $k = 1/6$)	A1 (6 marks)

Notes

1B1: cao

1M1: Use of arc length formula, need both $\sqrt{\ }$ and $\left(\dfrac{dy}{dx}\right)^2$.

1A1: $\int \cosh 3x\,dx$ cao

2M1: Attempt to integrate, getting a hyperbolic function o.e.

3M1: depends on previous M mark. Correct use of lna and 0 as limits. Must see some exponentials.

2A1: cao

Question Number	Scheme	Marks
3. (a)	$\overrightarrow{AC} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{BC} = -3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ $\overrightarrow{AC} \times \overrightarrow{BC} = 10\mathbf{i} - 15\mathbf{j} + 30\mathbf{k}$	B1, B1 M1 A1 (4)
(b)	Area of triangle ABC = $\frac{1}{2}\lvert 10\mathbf{i} - 15\mathbf{j} + 30\mathbf{k}\rvert = \frac{1}{2}\sqrt{1225} = 17.5$	M1 A1 (2)
(c)	Equation of plane is $10x - 15y + 30z = -20$ or $2x - 3y + 6z = -4$ So **r.** (2**i** – 3**j** + 6**k**) = -4 or correct multiple	M1 A1 (2) (8 marks)

Notes

a1B1: $\overrightarrow{AC} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ cao, any form

a2B1: $\overrightarrow{BC} = -3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ cao, any form

a1M1: Attempt to find cross product, modulus of one term correct.

a1A1: cao, any form.

b1M1: modulus of their answer to (a) – condone missing ½ here. To finding area of triangle by correct method.

b1A1: cao.

c1M1: [Using their answer to (a) to] find **equation** of plane. Look for **a.n** or **b.n** or **c.n** for p.

c1A1: cao

Question Number	Scheme	Marks
4(a)	$I_n = \left[x^n(-\tfrac{1}{2}\cos 2x)\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} -\tfrac{1}{2}nx^{n-1}\cos 2x\,dx$	M1 A1
	so $I_n = \left\langle \left[x^n(-\tfrac{1}{2}\cos 2x)\right]_0^{\frac{\pi}{4}} \right\rangle + \left[\tfrac{1}{4}nx^{n-1}\sin 2x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tfrac{1}{4}n(n-1)x^{n-2}\sin 2x\,dx$	M1 A1
	i.e. $I_n = \frac{1}{4}n\left(\frac{\pi}{4}\right)^{n-1} - \frac{1}{4}n(n-1)I_{n-2}$ *	A1cso
		(5)
(b)	$I_0 = \int_0^{\frac{\pi}{4}} \sin 2x\,dx = \left[-\tfrac{1}{2}\cos 2x\right]_0^{\frac{\pi}{4}} = \tfrac{1}{2}$	M1 A1
	$I_2 = \frac{1}{4}\times 2\times\left(\frac{\pi}{4}\right) - \frac{1}{4}\times 2\times I_0$, so $I_2 = \frac{\pi}{8} - \frac{1}{4}$	M1 A1 (4)
(c)	$I_4 = \left(\frac{\pi}{4}\right)^3 - \frac{1}{4}\times 4\times 3I_2 = \frac{\pi^3}{64} - 3\left(\frac{\pi}{8} - \frac{1}{4}\right) = \frac{1}{64}(\pi^3 - 24\pi + 48)$ *	M1 A1cso (2)

Notes

a1M1: Use of integration by parts, integrating $\sin 2x$, differentiating x^n.
a1A1: cao
a2M1: Second application of integration by parts, integrating $\cos 2x$, differentiating x^{n-1}.
a2A1: cao
a3A1: cso Including correct use of $\frac{\pi}{4}$ and 0 as limits.
b1M1: Integrating to find I_0 or setting up parts to find I_2.
b1A1: cao (Accept I_0 = ½ here for both marks)
b2M1: Finding I_2 in terms of π. If ‘n’’s left in M0
b2A1: cao
c1M1: Finding I_4 in terms of I_2 then in terms of π. If ‘n’’s left in M0
c1A1: cso

Question Number	Scheme	Marks
5. (a)	$\operatorname{ar}\sinh 2x, + x\dfrac{2}{\sqrt{1+4x^2}}$	M1A1, A1 (3)
(b)	$\therefore \int_0^{\sqrt{2}} \operatorname{arsinh}2x\mathrm{d}x = \left[x\operatorname{ar}\sinh 2x\right]_0^{\sqrt{2}} - \int_0^{\sqrt{2}} \dfrac{2x}{\sqrt{1+4x^2}}\mathrm{d}x$	1M1 1A1ft
	$= \left[x\operatorname{ar}\sinh 2x\right]_0^{\sqrt{2}} - \left[\dfrac{1}{2}(1+4x^2)^{\frac{1}{2}}\right]_0^{\sqrt{2}}$	2M1 2A1
	$= \sqrt{2}\operatorname{arsinh}2\sqrt{2} - [\frac{3}{2} - \frac{1}{2}]$	3DM1
	$= \sqrt{2}\ln(3+2\sqrt{2}) - 1$	4M1 3A1 (7) (10 marks)

Notes

a1M1: Differentiating getting an arsinh term **and** a term of the form $\dfrac{px}{\sqrt{1\pm qx^2}}$

a1A1: cao $\operatorname{ar}\sinh 2x$

a2A1: cao + $\dfrac{2x}{\sqrt{1+4x^2}}$

b1M1: rearranging their answer to (a). **OR** setting up parts
b1A1: ft from their (a) **OR** setting up parts correctly

b2M1: Integrating getting an arsinh or arcosh term **and** a $\left(1\pm ax^2\right)^{\frac{1}{2}}$ term o.e..

b2A1: cao

b3DM1: depends on previous M, correct use of $\sqrt{2}$ and 0 as limits.
b4M1: converting to log form.
b3A1: cao depends on all previous M marks.

Question Number	Scheme	Marks
6(a)	$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and so $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{xb^2}{ya^2}=-\frac{b\cos\theta}{a\sin\theta}$	M1 A1
	$\therefore\ y-b\sin\theta=-\frac{b\cos\theta}{a\sin\theta}(x-a\cos\theta)$	M1
	Uses $\cos^2\theta+\sin^2\theta=1$ to give $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$ ✳	A1cso (4)
(b)	Gradient of circle is $-\frac{\cos\theta}{\sin\theta}$ and equation of tangent is	M1
	$y-a\sin\theta=-\frac{\cos\theta}{\sin\theta}(x-a\cos\theta)$ or sets $a=b$ in previous answer	
	So $y\sin\theta+x\cos\theta=a$	A1 (2)
(c)	Eliminate x or y to give $y\sin\theta(\frac{a}{b}-1)=0$ or $x\cos\theta(\frac{b}{a}-1)=b-a$	M1
	l_1 and l_2 meet at $(\frac{a}{\cos\theta}, 0)$	A1, B1 (3)
(d)	The locus of R is part of the line $y = 0$, such that $x \geq a$ and $x \leq -a$ Or clearly labelled sketch. Accept "real axis"	B1, B1 (2) (11 marks)

Notes

a1M1: Finding gradient in terms of θ. Must use calculus.
a1A1: cao
a2M1: Finding equation of tangent
a2A1: cso (answer given). Need to get $\cos^2\theta+\sin^2\theta$ on the same side.
b1M1: Finding gradient and equation of tangent, **or** setting $a = b$.
b1A1: cao need not be simplified.
c1M1: As scheme
c1A1: $x=\frac{a}{\cos\theta}$, need not be simplified.
c1B1: $y = 0$, need not be simplified.
d1B1: Identifying locus as $y = 0$ or real/'x' axis.
d2B1: Depends on previous B mark, identifies correct parts of $y = 0$. Condone use of strict inequalities.

Question Number	Scheme	Marks
7(a)	$\text{f}(x) = 5\cosh x - 4\sinh x = 5\times\frac{1}{2}(e^x + e^{-x}) - 4\times\frac{1}{2}(e^x - e^{-x})$	M1
	$= \frac{1}{2}(e^x + 9e^{-x})$ $\ast$	A1cso (2)
(b)	$\frac{1}{2}(e^x + 9e^{-x}) = 5 \Rightarrow e^{2x} - 10e^x + 9 = 0$	M1 A1
	So $e^x = 9$ or 1 and $x = \ln 9$ or 0	M1 A1 (4)
(c)	Integral may be written $\int \frac{2e^x}{e^{2x}+9}\,\text{d}x$	B1
	This is $\frac{2}{3}\arctan\left(\frac{e^x}{3}\right)$	M1 A1
	Uses limits to give $\left[\frac{2}{3}\arctan 1 - \frac{2}{3}\arctan(\frac{1}{\sqrt{3}})\right] = \left[\frac{2}{3}\times\frac{\pi}{4} - \frac{2}{3}\times\frac{\pi}{6}\right] = \frac{\pi}{18}$ $\ast$	DM1 A1cso (5)
		(11 marks)

Notes

a1M1: Replacing both cosh$x$ and sinh$x$ by terms in e^x and e^{-x} condone sign errors here.
a1A1: cso (answer given)
b1M1: Getting a three term quadratic in e^x
b1A1: cao
b2M1: solving to $x =$
b2A1: cao need ln9 (o.e) and 0 (not ln1)
c1B1: cao getting into suitable form, may substitute first.
c1M1: Integrating to give term in arctan
c1A1: cao
c2M1: Depends on previous M mark. Correct use of ln3 and ½ ln3 as limits.
c2A1: cso must see them subtracting two terms in π.

Question Number	Scheme	Marks
8. (a)	$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 1 & 2-\lambda & 0 \\ -1 & 0 & 4-\lambda \end{vmatrix} = 0 \therefore (2-\lambda)(2-\lambda)(4-\lambda)-(4-\lambda)=0$	M1
	$(4-\lambda)=0$ verifies $\lambda=4$ is an eigenvalue (can be seen anywhere)	M1
	$\therefore (4-\lambda)\{4-4\lambda+\lambda^2-1\}=0 \;\; \therefore (4-\lambda)\{\lambda^2-4\lambda+3\}=0$	A1
	$\therefore (4-\lambda)(\lambda-1)(\lambda-3)=0$ and 3 and 1 are the other two eigenvalues	M1 A1 (5)
(b)	Set $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 4\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	M1
	Solve $-2x+y=0$ and $x-2y=0$ and $-x=0$ to obtain x = 0, y = 0, $z = k$	M1
	Obtain eigenvector as **k** (or multiple)	A1 (3)
(c)	l_1 has equation which may be written $\begin{pmatrix} 3+\lambda \\ 2-\lambda \\ -2+2\lambda \end{pmatrix}$	B1
	So l_2 is given by $\mathbf{r} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}\begin{pmatrix} 3+\lambda \\ 2-\lambda \\ -2+2\lambda \end{pmatrix}$	M1
	i.e. $\mathbf{r} = \begin{pmatrix} 8+\lambda \\ 7-\lambda \\ -11+7\lambda \end{pmatrix}$	M1 A1
	So $(\mathbf{r}-\mathbf{c})\times\mathbf{d}=\mathbf{0}$ where **c** = 8**i** + 7**j** - 11**k** and **d** =**i** - **j** + 7**k**	A1ft (5) (13 marks)

Notes

a1M1: Condone missing = 0. (They might expand the determinant using any row or column)

a2M1: Shows λ = 4 is an eigenvalue. Some working needed need to see = 0 at some stage.

a1A1: Three term quadratic factor cao, may be implicit (this A depends on 1st M only)

a2M1: Attempt at factorisation (usual rules), solving to λ = .

a2A1: cao. If they state λ = 1 and 3 please give the marks.

b1M1: Using A***x*** = 4***x*** o.e.

b2M1: Getting a pair of correct equations.

b1A1: cao

c1B1: Using **a** and **b**.

c1M1: Using r = M x their matrix in **a** and **b**.

c2M1: Getting an expression for l_2 with at least one component correct.

c1A1: cao all three components correct

c2A1ft: ft their vector, must have **r =** or **(r-c)x** d = 0 need both equation and r.